Course Syllabus

ISYE OMSA-6644

Simulation and Modeling for Engineering and Science

Spring 2026

(revised 11/17/25)

Professor: Dr. David Goldsman

Head Teaching Assistant: Briana Cope

Course Description

This course covers modeling of discrete-event dynamic systems and introduces simulation-based methods for using these models to solve engineering design and analysis problems.

Prerequisites

You will be expected to come in knowing a bit of basic calculus, probability, and statistics. But don't worry too much – we'll provide bootcamps on that material so as to make the class pretty much self-contained. In addition, this course will involve extensive computer programming, so it would be nice to have at least a little experience in something like Excel, just to bring back the programming memories.

Course Goals

- Learn how to develop simulation models and conduct simulation studies.
- Become familiar with the organization of simulation languages. In particular, we will do a great deal of modeling with Arena, a comprehensive simulation package with animation capabilities.
- Review statistical aspects including input analysis, random variate generation, output analysis, and variance reduction techniques.

Syllabus

- We make occasional changes to this wonderful syllabus.
- All nontrivial changes will be announced on Canvas.
- You are responsible for keeping up with any changes.

What to Expect in the Course

The course requires an above average amount of work, but none of the material is particularly difficult mathematically — there are simply a lot of topics that you'll need to digest quickly. We start out with calculus and prob/stats bootcamps with homeworks that are mostly extra credit (a good way to stick up on bonus points early in the semester). The simulation content is about 2/3 theory (heavy on prob/stats) and 1/3 programming (Arena + whatever your favorite

standard language happens to be, e.g., Python). The theory is necessary in order to establish a basis for the rigorous analyses that have to be undertaken in simulation applications. Students who desire more programming content and applications or who do not like the Arena simulation language can bide their time for a new course currently in development.

Grading Policies

- There will be two **midterms** and a **final exam**. Test questions are typically multiple choice or T/F.
- There will be 13 **Homework** assignments (not as bad as it sounds). There will be a bunch of bonus questions spread throughout the semester, which you can do to earn a few extra points. HWs that are late will suffer a 10*x*% deduction, where *x* is the number of days late, with *x* = 3 being the upper bound on the allowed lateness. So plan ahead!
- We will have a **project**, which you can select from among several theory- and applications-oriented topics. You will be allowed to work in small groups.
- **Turn Stuff in On-time!** Whether it's HWs, projects, or tests, **please** get things to us when you're supposed to. We give you very long windows and lots of warning, so you have plenty of time to plan ahead. Because our class is so large, *it will be very, very difficult to accommodate you if you miss a deadline.*
- You must achieve an overall weighted average of 60% to pass the course.
- Work hard and you will be rewarded – Grading is usually pretty generous. 😊
- Grading Disputes:
    - **Let's be winners, not whiners.** We are happy to discuss grades, but please make reasonable requests. 😊
    - To this end, we will generously provide various **bonus point opportunities** throughout the course; but because of this lovely act of kindness, we will not accept any test grade whining for matters involving up to 4 points. (Makes great sense, eh?!)

      Mother Teresa: "Wow, that's something nice that I would do!"

    - If you really, really want to request a regrade (for matters involving more than 4 points), simply fill out the convenient form that can be found at http://www.isye.gatech.edu/%7Esman/courses/gradegrovel.pdf.

- Grading Breakdown

Homework	10%
Project	10%
Midterm Exam 1	25%
Midterm Exam 2	25%
Final Exam	30%
Bonus Opportunities	2.5%
TOTAL	**102.5%**

Homework, Project, and Exam Due Dates

All homeworks, projects, and tests will be due at the times in the table at the end of this syllabus. These times are subject to change so please check back often. Please convert from Eastern Time (EDST/EST, Atlanta) to your local time zone using a Time Zone Converter.

Timing Policy

- The Modules follow a logical sequence, so they (mostly) need to be done in order.
- Homework Assignments and the Project should be completed by their due dates.
- Quizzes must be completed during the time allotted on the schedule.
- You will have access to the course content for the scheduled duration of the course.

Exam Policy

- For Quiz *x* (*x* = 1,2,3), you are allowed to use *x* sheets of paper, either 8.5"x11" or A4, with handwritten or printed notes (both sides of the sheet, 2*x* sides total).
- For all quizzes, you are allowed blank sheets of paper for scratch work. (All OMS Analytics and OMS CS students will be proctored; you will have to show the front and back of the blank sheets while you are being proctored.)
- You are also allowed to bring any reasonable calculator.
- You will **not** be allowed to use packages such as Excel, Arena, R, etc. during the exams.

Attendance Policy

- This is a fully online course.
- Login on a regular basis to complete your work, so that you do not have to spend a lot of time reviewing and refreshing yourself regarding the content.

Plagiarism Policy

- Plagiarism is considered a serious offense. You are not allowed to copy and paste or submit materials created or published by others, as if you created the materials. All materials submitted and posted must be your own.
- These days, there is a lot of buzz regarding ChatGPT (and related applications). We suppose that you can use ChatGPT to help you with very preliminary versions of some of your written verbiage, but (i) *do not let ChatGPT do your thinking for you*, and (ii) *make sure to properly and thoroughly cite your use of ChatGPT*.

Student Honor Code

All GT students should abide by the Georgia Tech Student Honor Code.

- Review the Georgia Tech Student Honor Code: https://osi.gatech.edu/content/honor-code
- You are responsible for completing your own work.
- Any GT student suspected of behavior in violation of the Georgia Tech Honor Code will be referred to Georgia Tech's Office of Student Integrity.

Communication

- Feel totally free to contact your instructor, teaching assistants, and fellow learners via the Piazza discussion forums. Our Piazza access code is **sp26_isye6644**. *(Please, please, please do not email the instructor directly unless it's something really, really important.)*

Often, discussions with fellow learners are the sources of key pieces of learning and are often funny and entertaining. Some suggestions:

- Always be courteous and nice (see Netiquette below).
- Please make sure that your subject line PRECISELY states what problem you are asking about, as failure to do so causes everyone a great deal of time trying to figure out what you need. For instance, "Fall 2023 Practice Test 3, Question 5a".
- Your problem may have already been addressed! So make sure that you sniff around the Piazza forum to see if that's the case! That avoids repetitive and redundant and repetitive repetition.
- Think about your problem a bit and give it the old college try before asking about it on Piazza. Don't give up too early before you punt!

Netiquette

- Netiquette refers to etiquette that is used when communicating on the Internet. Review the Core Rules of Netiquette. When you are communicating via email, discussion forums or synchronously (in real-time), please use correct spelling, punctuation, and grammar consistent with the academic environment and scholarship[1].
- We expect all participants in Georgia Tech's MS in Analytics program, (learners, faculty, teaching assistants, staff) to interact respectfully. *You must always play nice with your fellow students and dedicated TAs.* Learners who do not adhere to this guideline may be removed from the course.

[1]Conner, P. (2006–2014). Ground Rules for Online Discussions, Retrieved 8/14/2020 from https://tilt.colostate.edu/TipsAndGuides/Tip/128

Course Materials

- All content and course materials can be accessed online.
- There is no required textbook for this course, though students are encouraged to find copies of the following references:
  - Law, A. M., *Simulation Modeling and Analysis*, 6th edition, McGraw-Hill Education, New York, 2024. [This textbook is most for the "theory" aspects of the course.]
  - Kelton, W. D., Zupick, N. B., and Ivey, N., *Simulation with Arena*, 7th edition, McGraw-Hill, New York, 2024. [This book covers the Arena simulation language.]
  - If you want to review probability and statistics, you go to my ISYE 6739 course website www.isye.gatech.edu/~sman/courses/6739 for complete course materials; and/or you can (i) get a free copy of my wonderful book, *A First Course in Probability and Statistics*, from our ISYE 6644 Canvas site, or (ii) buy an el cheapo softbound version here (just click). We've heard that this makes a great Groundhog Day gift for the whole family!

Technology/Software Requirements

- Internet connection (DSL, LAN, or cable connection desirable)
- R statistical software (free download; see cran.r-project.org)
- Arena simulation software

    - Arena is free! Get it here (but make sure to click the "Student" option on the "Job Type" menu)!
    - *Arena requires a Windows operating system* to run on your computer.
    - If you don't have Windows, you can run Arena thru ISyE's Virtual Lab. To get an account, see https://www.isye.gatech.edu/academics/doctoral/current-students/computing#services
    - Arena (and our corresponding lecture material) is currently transitioning to a new version, so the latest Arena version doesn't perfectly match what's in the notes. The good news is that everything still works. ☺
- Adobe Acrobat PDF reader (free download; see https://get.adobe.com/reader/)

Disabilities and Special Circumstances

- If you have an issue requiring special accommodations, please make an appointment with the Office of Disability Services to discuss the appropriate procedures. Their website is http://disabilityservices.gatech.edu
- In some cases, religious observances or other events may conflict with scheduled class activities. *In such situations students can be given an alternative means of meeting the academic requirement.* **Students must notify the instructor of any such conflicts, with the specific dates, within the first two weeks of classes.** Students requiring disability accommodations are also requested to make arrangements with the instructor, within the same period if possible.
- We will work with you if you come down with any significant illness. Potential solutions include granting small time extensions, re-weighting test scores if you happen to miss one, and even giving an incomplete (as a last resort).

Course Topics and Pacing Schedule

The table below contains a course topic outline and homework due dates. [Note that some topics below are marked as **OPTIONAL**. You will not be given mandatory homework nor will you be tested on those topics; but we have nevertheless included this material in case you need additional review or would like to delve into a topic further.]

Weeks	Course Topics	Release Dates (all times EDT/EDST)
Week 1 (Jan 12–16)	**Module 1: Whirlwind Tour of Simulation** Lesson 1: Getting to Know You Lesson 2: Syllabus Lesson 3: Whirlwind Tour Lesson 4: Whirlwind Tour – History Lesson 5: What Can We Do for You Lesson 6: Some Baby Examples Lesson 7: More Baby Examples Lesson 8: Generating Randomness Lesson 9 **[OPTIONAL]**: Simulation Output Analysis	M Jan 12 at 8:00 a.m.
Week 1 Homework	Homework 1	F Jan 16 at 8:00 a.m. – F Jan 23 at 11:59 p.m.
Week 2 (Jan 19–23) MLK Day Jan 19	**Module 2: Bootcamps** Lesson 1 **[OPTIONAL]**: Calculus Primer Lesson 2 **[OPTIONAL]**: Saved By Zero! Solving Equations Lesson 3 **[OPTIONAL]**: Integration Lesson 4 **[OPTIONAL]**: Integration Computer Exercises Lesson 5: Probability Basics Lesson 6: Simulating Random Variables Lesson 7: Great Expectations Lesson 8: Functions of a Random Variable Lesson 9: Jointly Distributed Random Variables	M Jan 19 at 8:00 a.m.
Week 2 Town Hall	Meet the Prof and TAs! https://gatech.zoom.us/j/94383992580	Tu Jan 20 at 7:00 p.m.
Week 2 Homework	Homework 2	F Jan 23 at 8:00 a.m. – F Jan 30 at 11:59 p.m.
Week 3 (Jan 26–30)	**Module 2 (cont): Some More Bootcamps** Lesson 10 **[OPTIONAL]**: Conditional Expectation Lesson 11: Covariance and Correlation	M Jan 26 at 8:00 a.m.

	Lesson 12: Probability Distributions Lesson 13: Limit Theorems Lesson 14 **[OPTIONAL]**: Introduction to Estimation Lesson 15 **[OPTIONAL]**: Maximum Likelihood Estimation Lesson 16 **[OPTIONAL]**: Confidence Intervals	
Week 3 Homework	Homework 3	F Jan 30 at 8:00 a.m. – F Feb 6 at 11:59 p.m.
Week 4 (Feb 2–6) Happy Groundhog Day! Happy Groundhog Day! Happy Groundhog Day! Happy Groundhog Day!	**Module 3: Hand Simulations** Lesson 1: Stepping Through Differential Equation Lesson 2: Monte Carlo Integration Lesson 3: Monte Carlo Integration Demo Lesson 4: Making Some Pi Lesson 5: A Single-Server Queue Lesson 6: An (*s*,*S*) Inventory System Lesson 7: An (*s*,*S*) Inventory System Demo Lesson 8: Simulating Random Variables Lesson 9: Simulating Random Variables Demo Lesson 10: Spreadsheet Simulation	M Feb 2 at 8:00 a.m.
Week 4 Homework	Homework 4	F Feb 6 at 8:00 a.m. – F Feb 13 at 11:59 p.m.
Week 5 (Feb 9–13) Feb 14: Happy Valentine's Day! ❤️	**Module 4: General Simulation Principles** Lesson 1: Steps in a Simulation Study Lesson 2: Some Useful Definitions Lesson 3: Time-Advance Mechanisms Lesson 4: Two Modeling Approaches Lesson 5: Simulation Languages	M Feb 9 at 8:00 a.m.
Project Milestone 1	**Project Topics Announced**	M Feb 9 at 8:00 a.m. **Topic and group selection** due F **Feb 20** at 11:59 p.m.
Week 5 Homework	Homework 5	F Feb 13 at 8:00 a.m. – F Feb 20 at 11:59 p.m.
Week 6 (Feb 16–20)	**Module 5: The Arena Simulation Language** Lesson 1: Introduction Lesson 2: Process-interaction	M Feb 16 at 8:00 a.m.

	Lesson 3: Let's Meet Arena! Lesson 4: The Arena Basic Template Lesson 5: Create-Process-Dispose Modules Lesson 6: The Process Module Lesson 7: Resource, Schedule, and Queue Spreadsheets Lesson 8: The Decide Module Lesson 9: The Assign Module Lesson 10: Attribute, Variable, and Entity Spreadsheets Lesson 11: Arena Internal Variables Lesson 12: Displaying Stuff Lesson 13: Batch, Separate, and Control Lesson 14: Run Setup and Control	
Week 6 Town Hall	Meet the Prof and TAs! Chat Session for Exam 1 https://gatech.zoom.us/j/94383992580	Tu Feb 17 at 7:00 p.m.
Week 6 Homework	Homework 6	F Feb 20 at 8:00 a.m. – F Feb 27 at 11:59 p.m.
Midterm Exam 1	**Midterm Exam 1 [Covers everything up to and including Lesson 9 from Week 6. See Topics Attachment.]**	F Feb 20 at 8:00 a.m. – **Su Feb 29** at 11:59 p.m.
Week 7 (Feb 23–27)	**Module 5 (cont.): More Arena** Lesson 15: Two-Channel Manufacturing Example Lesson 16: Fake Customers Lesson 17: The Advanced Process Template Lesson 18: Resource Failures + Maintenance Lesson 19: The Blocks Template Lesson 20: The Joy of Sets Lesson 21: Description of Call Center Lesson 22: Call Center Demo Lesson 23: An Inventory Model Lesson 24: One Line vs Two Lines? Lesson 25: A Re-entrant Queue Lesson 26: SMARTS Files and Rockwell Demos Lesson 27: A Manufacturing System Demo	M Feb 23 at 8:00 a.m.
Week 7 Homework	Homework 7	F Feb 27 at 8:00 a.m. – F Mar 6 at 11:59 p.m.
Week 8 (Mar 2–6)	**Module 5 (cont.): Still More Arena** *The following lessons are **NOT OPTIONAL** (in spite of what the videos may say)!* Lesson 28: Mfg System Details: Advanced Transfer Panel	M Mar 2 at 8:00 a.m.

	Lesson 29: Mfg System Details: Sequences Lesson 30: Mfg System Details: Advanced Sets Lesson 31: Mfg System Details: Model Walk-Through Lesson 32: Mfg System Details: Transporters and Conveyors **Module 6: Random Number Generation** Lesson 1: Introduction Lesson 2: Some Lousy Generators Lesson 3: Linear Congruential Generators Lesson 4: Tausworthe Generators Lesson 5: Generalization of LCGs Lesson 6: Choosing a Good Generator – Some Theory Lesson 7: Choosing a Good Generator – Statistics Tests, Intro Lesson 8: Choosing a Good Generator – Goodness-of-Fit Tests Lesson 9: Choosing a Good Generator – Independence Tests I Lesson 10: **[OPTIONAL]**: Independence Tests II	
Week 8 Homework	Homework 8	F Mar 6 at 8:00 a.m. – F Mar 13 at 11:59 p.m.
Week 9 (Mar 9–13) Mar 14: Happy π Day!	**Module 7: Random Variate Generation** Lesson 1: Introduction Lesson 2: Inverse Transform Method Lesson 3.1: ITM – Continuous Examples Lesson 3.2: ITM – Continuous Examples DEMO 1 Lesson 3.3: ITM – Continuous Examples DEMO 2 Lesson 4: Inverse Transform Method - Discrete Examples Lesson 5 **[OPTIONAL]**: ITM – Empirical Distributions Lesson 6.1: Convolution Method Lesson 6.2: Convolution Method DEMO Lesson 7: Acceptance-Rejection Method Lesson 8 **[OPTIONAL]**: Proof of the A-R Method Lesson 9.1: A-R Method – Continuous Examples Lesson 9.2: A-R Method – Continuous Examples DEMO Lesson 10: A-R Method – Poisson Distribution	M Mar 9 at 8:00 a.m.
Project Milestone 2	**Project Progress Report** (nice and simple; rubric TBA)	Due F Mar 13 at 11:59 p.m.
Week 9	Homework 9	F Mar 13 at 8:00 a.m. – F Mar 20 at 11:59 p.m.
Week 10 (Mar 16–20)	**Module 7 (cont.): More RV Generation** Lesson 11: Composition Lesson 12: Box-Muller Normal RVs Lesson 13: Order Statistics Other Stuff	M Mar 16 at 8:00 a.m.

	Lesson 14: Multivariate Normal Distribution Lesson 15: Baby Stochastic Processes Lesson 16.1: Nonhomogeneous Poisson Processes Lesson 16.2: Nonhomogeneous Poisson Processes DEMO Lesson 17.1: Time Series Lesson 17.2: Time Series DEMO Lesson 18: Queueing Lesson 19.1: Brownian Motion Lesson 19.2: Brownian Motion DEMO	
Week 10 Town Hall	Meet the Prof and TAs! Chat Session for Exam 2 https://gatech.zoom.us/j/94383992580	Tu Mar 17 at 7:00 p.m.
Week 10 Homework	Homework 10	F Mar 20 at 8:00 a.m. – F Apr 3 at 11:59 p.m.
SPRING BREAK! (Mar 23–27)	**Note that you still have to get stuff turned in on April 3, so make sure to plan ahead!!**	
Midterm Exam 2	**Midterm Exam 2 [Covers everything up to and including Lesson 13 from Week 10, with emphasis on more-recent stuff. See Topics Attachment.]**	F Mar 20 at 8:00 a.m. – **Su Apr 5** at 11:59 p.m.
Week 11 (Mar 30 – Apr 3)	**Module 8: Input Analysis** Lesson 1: Introduction Lesson 2: Identifying Distributions Lesson 3: Unbiased Point Estimation Lesson 4: Mean Squared Error Lesson 5: Maximum Likelihood Estimators Lesson 6: MLE Examples Lesson 7: Invariance Property of MLEs Lesson 8 **[OPTIONAL]**: The Method of Moments Lesson 9: Goodness-of-Fit Tests Lesson 10: Exponential Example Lesson 11: Weibull Example Lesson 12: Still More Goodness-of-Fit Tests Lesson 13: Problem Children Lesson 14: Demo Time	M Mar 30 at 8:00 a.m.
Week 11 Homework	Homework 11	F Apr 3 at 8:00 a.m. – F Apr 10 at 11:59 p.m.
Week 12 (Apr 6–10)	**Module 9: Output Analysis** Lesson 1: Introduction Lesson 2: Mathematical Interlude **[Don't Panic!]**	M Apr 6 at 8:00 a.m.

	Lesson 3: Finite-Horizon Analysis Lesson 4: Finite-Horizon Extensions Lesson 5: Simulation Initialization Issues Lesson 6: Steady-State Analysis Lesson 7: Properties of Batch Means Lesson 8: Other Steady-State Methods	
Week 12 Homework	Homework 12	F Apr 10 at 8:00 a.m. – F Apr 17 at 11:59 p.m.
Week 13 (Apr 13–17)	**Module 10: Comparing Systems** Lesson 1: Introduction Lesson 2: Confidence Interval for the Mean Lesson 3: CIs for the Difference in Two Means Lesson 4: Paired CI for the Difference in Two Means Lesson 5: CIs for the Mean Difference in Simulations Lesson 6: Common Random Numbers Lesson 7: Antithetic Random Numbers Lesson 8 **[OPTIONAL]**: Control Variates Lesson 9: Ranking and Selection Methods	M Apr 13 at 8:00 a.m.
Week 13 Homework	Homework 13	F Apr 17 at 8:00 a.m. – F Apr 24 at 11:59 p.m.
Weeks 14 + 15 (Apr 20–Apr 28)	**Module 10 (cont.): More Comparing Systems** Lesson 10: Normal Means Selection Lesson 11: Single-Stage Normal Means Procedure Lesson 12: Normal Means Extensions Lesson 13: Bernoulli Probability Selection Lesson 14: Bernoulli Extensions Lesson 15: Multinomial Cell Selection Lesson 16: Multinomial Procedure + Extensions Lesson 17: Summary	M Apr 20 at 8:00 a.m.
Weeks 14 + 15 Homework	No HW!	
Project Milestone 3	**Project Due**	M Apr 20 at 8:00 a.m. **due Tu Apr 28** at 11:59 p.m.
Week 14 Town Hall	Meet the Prof and TAs! Chat Session for Final exam https://gatech.zoom.us/j/94383992580	Tu Apr 21 at 7:00 p.m.

Final Exam	**Final Exam [Covers everything, with emphasis on more-recent stuff. See Topics Attachment below.]**	Th Apr 30 at 8:00 a.m. – **Th May 7** at 11:59 p.m.
CIOS Survey	**If 80% of the class turns in their CIOS survey, then I'll add 1% to everyone's final score! Wow!**	**Th Aug 6**

Test Topix for ISyE 6644, Spring 2026

- We've tried my best to make this list as complete as possible, but we may have missed a topic or two. That being said, you are responsible for everything that we do in class or homework.
- As GT students, you are expected to formulate problems and solution strategies which are more than mere rote regurgitation of material you learned in class. Thus, you shouldn't be surprised if some questions cover natural extensions of material from class.
- We'll supply all necessary tables, e.g., N(0,1), t, and χ^2, but you can feel free to use your own.

TEST 1 TOPIX

1. **Intro Material**
   a. Definition of simulation
   b. Advantages and disadvantages of simulation
   c. History of simulation
   d. Typical questions and applications

2. **Calculus, Probability, and Statistics Review**
   a. **Calculus [not really responsible for this material, except I might make you search for a zero]**
      i. Basic definitions
      ii. Derivatives
      iii. Solving for zeros
      iv. Integration
      v. Numerical integration
   b. Probability Preliminaries
      i. Conditional probability
      ii. Independent events
      iii. Definition of random variable
      iv. Discrete RV's and probability mass function
      v. Continuous RV's and probability density function
      vi. Cumulative distribution function
   c. Simulating RV's (first pass)
      i. Discrete uniform distribution
      ii. General discrete distribution
      iii. Inverse Transform Theorem for continuous RV's
      iv. Exponential (and other) continuous distributions via IVT.

v. Generating U(0,1)'s via desert island algorithm, including walk-through of pseudo-code.

d. Expected Values
   i. Definition
   ii. Discrete and continuous examples of expected value
   iii. Law of the Unconscious Statistician
   iv. Moments, central moments, variance, standard deviation
   v. Discrete and continuous examples of LOTUS
   vi. Moment generating function
   vii. Examples and properties of mgf's
e. Functions of a RV
   i. Discrete examples
   ii. Continuous examples
   iii. IVT methods (again) with examples
   iv. Relationship with LOTUS
f. Jointly distributed RV's
   i. Definition of joint cdf
   ii. Marginal cdf's
   iii. Joint and marginal pmf's
   iv. Joint and marginal pdf's
   v. Examples for discrete and continuous cases
   vi. Independent RV's
   vii. Conditional pmf's and pdf's
   viii. **Conditional expectation [this won't be on the test]**
   ix. **Double expectation E(E(Y|X)) = EY, including examples [this won't be on the test]**
g. Covariance and correlation
   i. Definitions
   ii. Relationship between independence and correlation
   iii. Examples
   iv. Miscellaneous properties (e.g., Var(X+Y), bounds on correlation, etc.)
h. Probability distributions
   i. Discrete distributions
      1. Bernoulli
      2. Binomial
      3. Geometric
      4. Poisson (including discussion on Poisson processes)
   ii. Continuous distributions
      1. Uniform
      2. Exponential (including memoryless property)
      3. Erlang, Gamma distributions

4. Triangular
5. Normal (including Standard Normal)
6. Other sampling distributions (including chi-square, t, F, and various relationships with each other)

i. Limit theorems
   i. Linear combinations of independent normal (including distribution of sample mean)
   ii. Convergence in distribution
   iii. Law of Large Numbers
   iv. Central Limit Theorem for independent and identically distributed data.
   v. Examples

j. Statistics Tidbits **[this material will eventually be covered in modules 8, 9, 10, so it's in fair territory to have it on the final! ☺]**
   i. Properties of sample mean and sample variance
   ii. Confidence intervals for the mean and variance

3. Hand Simulations

a. "Simulating" a differential equation
b. Monte Carlo integration
c. Determining π via simulation (dart tossing on a circle and sphere)
d. Single-server queue (including FIFO and LIFO service disciplines)
e. (s,S) inventory system
f. Simulating RV's (repeats some material from the Prob/Stats review)
g. Spreadsheet simulation (e.g., stock portfolio in Excel)

4. General Simulation Principles

a. Steps in a simulation study
b. List of various simulation definitions (e.g., event, system state, simulation clock, etc.)
c. Event-Scheduling vs. Process Interaction modeling approaches
d. How are events processed?
e. Future events list + extended example
f. Simulation languages – what to look for

Plus, the first few Arena mini-topix below…

5. Arena

a. Layout of Arena screen (panels, modules, etc.)
b. Basic Process template: CREATE-PROCESS-DISPOSE modules
c. SEIZE-DELAY-RELEASE inside of the PROCESS module.
d. Resource, Schedule, Queue, Entity, and other spreadsheets
e. DECIDE module – probabilistic and conditional routing
f. ASSIGN module

TEST 2 TOPIX

Everything from Test 1 + the following (with less emphasis placed on the Test 1 material)…

g. Simple examples, e.g. (partial list),
    i. Single-server queue
    ii. Parallel servers
    iii. Schedules for servers
    iv. Multiple arrival streams
h. Displays, graphics, etc.
i. BATCH and SEPARATE modules
j. Run set-up and control
k. More-sophisticated queueing networks (e.g., two-channel manufacturing example, call center example)
    i. Advanced Process modules (e.g., SEIZE, DELAY, RELEASE modules)
    ii. Some primitive blocks (e.g., QUEUE)
    iii. Use of "pretend" customers
    iv. Nonhomogeneous Poisson arrivals
    v. Use of resource sets, including how to prioritize servers
    vi. Use of submodels
l. Inventory processes
m. Crazy examples such as re-entrant queues
n. SMARTS files and other Rockwell examples
o. Manufacturing systems
    i. Advanced Transfer modules (e.g., ROUTE, ENTER, LEAVE)
    ii. Sequences of customer visitation locations
    iii. Advanced sets of sequences
    iv. Transporters and conveyors

6. Uniform Random Number Generation

a. Overview – desirable properties of a pseudo-random number generator
b. Some generators we won't use, e.g.,
    i. PRN's from tables
    ii. Midsquare
c. Linear congruential generators
    i. Cycling
    ii. 16807 desert island generator (again)
    iii. RANDU (a bad generator)
d. Tausworthe generator
e. Combined generators

i. L'Ecuyer's generator of cycle length 2^{191}
ii. Mersenne Twister

f. Some theoretical considerations, e.g., from Knuth's book
g. Statistical tests for randomness
    i. Goodness-of-fit test – Chi-squared
    ii. Runs tests for independence
        1. Runs up and down
        2. Runs above and below the mean
        3. **Autocorrelation test [this won't be on the test]**

7. Random Variate Generation

a. Inverse Transform Theorem (yet again)
    i. Proof
    ii. Discrete example adaptations
    iii. Continuous examples
        1. East ones such as Exponential, Weibull, etc.
        2. Slightly harder examples such as Triangle distribution
        3. Normal distribution, both exact and approximate methods
    iv. Special case methods, e.g., Geometric
    v. **Empirical distributions [this won't be on the test]**
b. Convolution method
    i. Binomial
    ii. Triangle
    iii. Erlang
    iv. CLT
        1. Desert island sum of Uniforms to generate Normal
        2. Normal approximation to Poisson (including continuity correction)
    v. Cauchy
        1. Cauchy's add up to another Cauchy
        2. IVT method
        3. Ratio of two Normals
c. Acceptance-Rejection methods
    i. Trivial Uniform example
    ii. Some discussion on general method
    iii. **Proof of the general method [don't expect to see this on the test, unless I'm in a really bad mood!]**
    iv. Examples involving polynomial and half-normal p.d.f.'s
    v. Poisson distribution
d. Composition
e. Special-case techniques
    i. Box-Muller method for Normal distribution

ii. Extensions of B-M, e.g., Cauchy, Chi-squared.
iii. Generating min's and max's of iid RV's, e.g., min of iid Exponentials.

TEST 3 (FINAL) TOPIX

Everything in the freaking course, including the following new stuff (with less emphasis placed on the Tests 1 and 2 material)…

f. Multivariate Normal
   i. Definition in 2 and then >2 dimensions
   ii. Cholesky decomposition method for generating realizations (exact expression in 2 dimensions, algorithm for >2 dimensions)
g. Stochastic processes
   i. Markov chains
   ii. Poisson processes
   iii. Nonhomogeneous Poisson processes (via thinning method)
   iv. Time series
      1. MA(1)
      2. AR(1)
      3. EAR(1)
      4. ARTOP
   v. M/M/1 queue waiting times
   vi. Brownian motion
      1. Definition and history
      2. Elementary properties, including covariance structure
      3. General CLT
      4. How to generate
      5. Geometric BM and financial applications

8. Input Analysis

a. General discussion
   i. Careful about GIGO with respect to simulation input
   ii. What makes a good distribution
   iii. Identification of obvious distributions
b. Estimation review
   i. Unbiased estimators
      1. Definition
      2. Sample mean
      3. Sample variance
      4. Other examples such as Unif(0,θ)

ii. Mean squared error
iii. Maximum likelihood estimators
    1. Definition
    2. Examples such as Exponential
    3. Two-dimensional examples such as Normal with unknown mean and variance
    4. Other tougher examples such as Unif(0,θ) and Gamma
    5. Invariance Property + examples
iv. Method of Moments [this won't be on the test (sorry, MoM!)]

c. Goodness-of-fit tests for input distributions
    i. Chi-squared for Exponential
    ii. Chi-squared for Weibull, including search techniques such as bisection and Newton
    iii. Kolmogorov-Smirnov
    iv. More goodness-of-fit tests
d. Problem Children
    i. Little or no data
    ii. Data from an unusual distribution
    iii. Nonstationary data
    iv. Multivariate / correlated data
e. Arena Input Analyzer demo

9. Output Analysis

a. Introduction
    i. The need for output analysis in a proper statistical study
    ii. Simulation data isn't iid normal, and this is a problem
    iii. Types of output analysis – finite-horizon (terminating) and steady-state
b. A mathematical interlude related to the fact that the variance of the sample mean isn't Var(X_i)/n, and its consequences **[I might ask you to calculate the variance of the sample mean for a specific process, but nothing else.]**
c. Finite-horizon (terminating) simulations
    i. Examples
    ii. Confidence intervals for mean performance via the method of Independent Replications
d. Initialization problems
e. Steady-state analysis for a single system
    i. Examples
    ii. Confidence intervals for the steady-state mean via the method of Batch Means
    iii. Properties of Batch Means **[The more-mathematical aspects of this topic this won't be on the test.]**

iv. Overlapping Batch Means
v. Other methods

10. Comparing Systems

a. Classical confidence interval for the mean of one normal population
b. Classical confidence interval to compare the means of two normal systems
    i. Variance completely unknown
    ii. Paired-t CI
    iii. Use in simulation scenarios
c. Variance reduction techniques
    i. Common random numbers
    ii. Antithetic random numbers
    iii. **Control variates [this won't be on the test]**
d. Ranking and selection methods to compare means of >2 systems
    i. Definition of problem
    ii. Relevance to simulation
    iii. Indifference-zone approach
    iv. Normal means selection problem
        - Bechhofer's single-stage procedure
        - Extensions
    v. Bernoulli parameter selection problem
        - Sobel and Huyett single-stage procedure
        - Extensions
    vi. Multinomial cell selection problem
        - Multinomial review and motivation
        - Bechhofer, Elmaghraby, and Morse single-stage procedure
        - Extensions